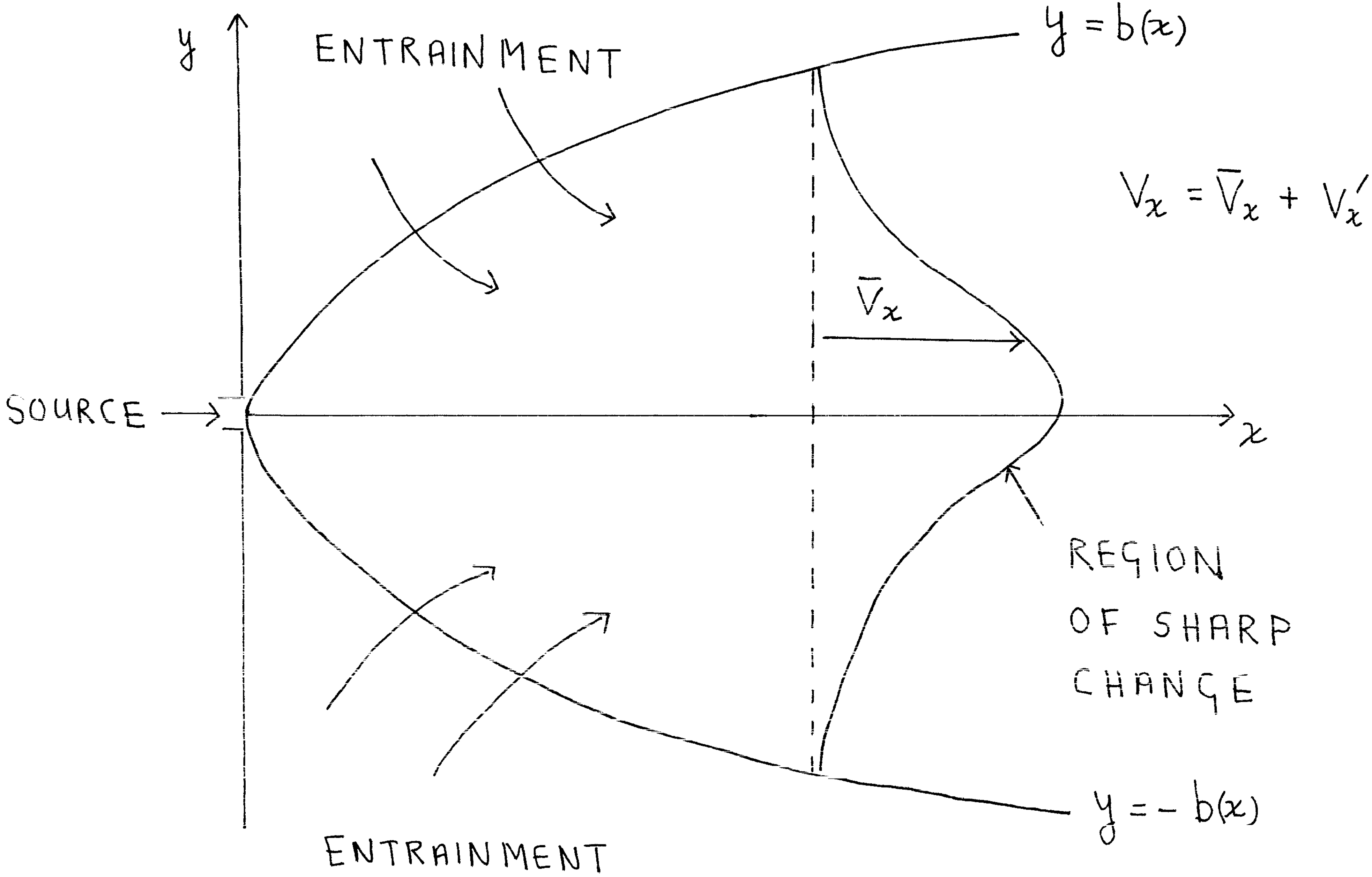
TURBULENT TWO-DIMENSIONAL JET

DAVID MASON

ERICK MUBAI

UNIVERSITY OF THE WITWATERSRAND

MODEL OF SNEEZE



- TURBULENT FLOW
- BOUNDARY LAYER THEORY
- TURBULENT TWO-DIMENSIONAL JET
- ANALYTICAL SOLUTION
- MODEL OF A SNEEZE

- TURBULENT FLOW

MEAN FLOW + FLUCTUATION

$$V_i(x,y,t) = \bar{V}_i(x,y) + V_i'(x,y,t)$$

$$\bar{\bar{V}}_i = \bar{V}_i \quad , \quad \overline{V_i'} = 0 \quad , \quad \overline{V_i' V_j'} \neq 0$$

NAVIER-STOKES EQUATION

$$\rho \left(\frac{\partial V_x}{\partial t} + \underset{\substack{\uparrow \\ \text{NONLINEAR TERM}}}{V_x \frac{\partial V_x}{\partial x}} + V_y \frac{\partial V_x}{\partial y} \right) = - \frac{\partial p}{\partial x} + \mu \left(\frac{\partial^2 V_x}{\partial x^2} + \frac{\partial^2 V_x}{\partial y^2} \right)$$

THE REYNOLDS STRESSES $\quad \bar{\tau}_{xy}^{(T)} = - \rho \overline{V_x' V_y'} \neq 0$

BOUSSINESQUE HYPOTHESIS $\quad -\rho \overline{v_i' v_j'} = \mu_T \left(\frac{\partial \bar{v}_i}{\partial x_j} + \frac{\partial v_j'}{\partial x_i} \right)$

EDDY VISCOSITY $\quad \mu_T$

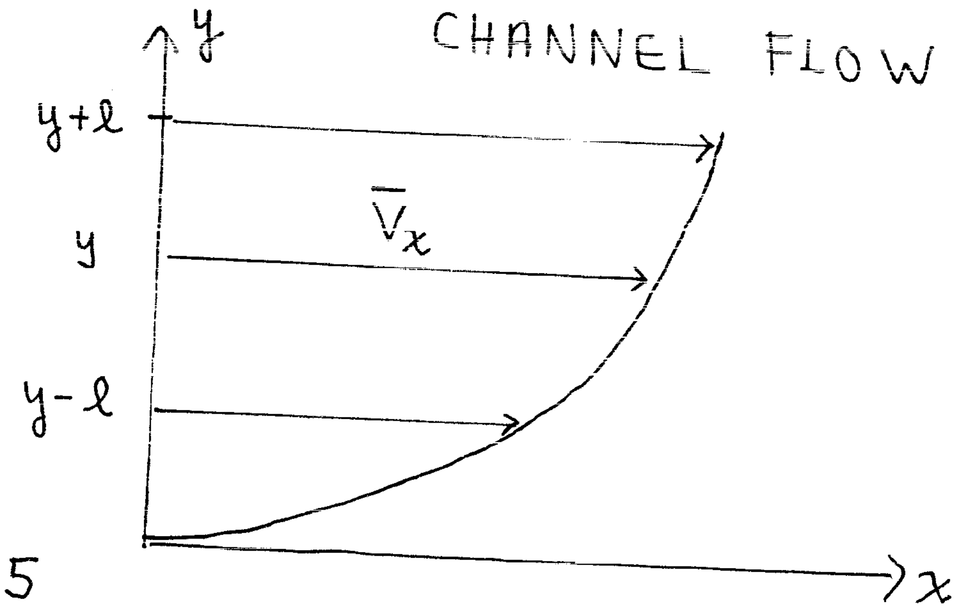
PROPERTY OF THE FLOW

TURBULENT FLOW $\quad \mu$ REPLACED BY $\mu + \mu_T$

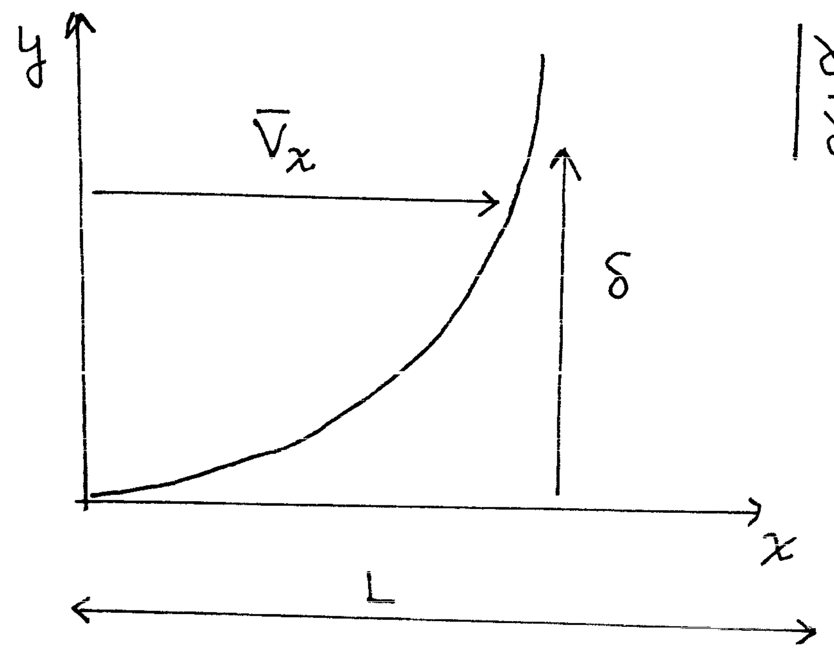
PRANDTL'S MIXING LENGTH MODEL

$$\mu_T = \rho \, l^2(x) \left| \frac{\partial \bar{V}_x}{\partial y} \right|$$

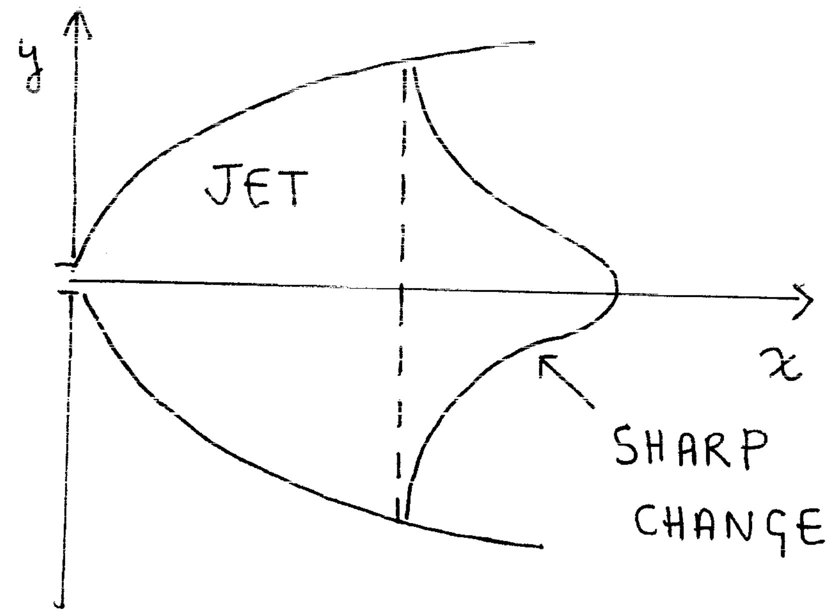
$$\nu_T = \frac{\mu_T}{\rho} = l^2(x) \left| \frac{\partial \bar{V}_x}{\partial y} \right|$$



• BOUNDARY LAYER THEORY



$$\left|\frac{\partial \bar{V}_x}{\partial y}\right| \gg \left|\frac{\partial \bar{V}_x}{\partial x}\right|$$



REYNOLDS NUMBER

$$Re = \frac{UL}{\nu}$$

$$Re(T) = \frac{UL}{\nu + \nu_T}$$

BOUNDARY LAYER PROVIDED

$$\frac{\delta}{L} = \frac{1}{\sqrt{Re(T)}} \ll 1$$

$$\sqrt{Re(T)} \gg 1$$

APPLIES TO

SOLID BOUNDARY
JET
WAKE
PLUME

- TURBULENT JET

REYNOLDS AVERAGED EQUATIONS

$$\bar{V}_x \frac{\partial \bar{V}_x}{\partial x} + \bar{V}_y \frac{\partial \bar{V}_x}{\partial y} = \frac{\partial}{\partial y}\left[(\nu + \nu_T) \frac{\partial \bar{V}_x}{\partial y} \right]$$

$$\nu_T = \ell^2(x) \left| \frac{\partial \bar{V}_x}{\partial y} \right|$$

CONSERVATION OF MASS

$$\frac{\partial \bar{V}_x}{\partial x} + \frac{\partial \bar{V}_y}{\partial y} = 0 \; ; \qquad \bar{V}_x = \frac{\partial \psi}{\partial y}, \qquad \bar{V}_y = -\frac{\partial \psi}{\partial x}$$

BOUNDARY CONDITIONS

$$V_y(x,0) = 0, \quad \frac{\partial \bar{V}_x}{\partial y}(x,0) = 0, \quad \bar{V}_x(x, b(x)) = 0, \quad \frac{\partial \bar{V}_x}{\partial y}(x, b(x)) = 0.$$

CONSERVED QUANTITY

$$J = 2\rho \int_0^{b(x)} \bar{V}_x^2(x,y)\,dy$$

$= $ CONSTANT INDEPENDENT OF x

$J = $ MEAN MOMENTUM FLUX

- SIMILARITY SOLUTION

SCALING TRANSFORMATION

$$\bar{x} = \lambda^a x, \quad \bar{y} = \lambda^b y, \quad \bar{\psi} = \lambda^c \psi, \quad \bar{\ell} = \lambda^m \ell$$

- REDUCE PDE TO ODE

- ANALYTICAL SOLUTION WHEN $\nu = 0$ $(\nu \ll \nu_T)$

- MODEL OF A SNEEZE

FIT PARAMETER VALUES FOR A SNEEZE

RANGE OF JET IN DOWNSTREAM X-DIRECTION

RANGE OF JET IN TRANSVERSE Y-DIRECTION

ESTIMATES OF SOCIAL DISTANCE

MODEL EFFECT OF MASK BY CHANGING VALUE OF J
